The Unified Ontology

A Compendium on Gravitation, the Particle Spectrum, and Soliton Configurations via Spectral Theory, Orbital Angular Momentum, and Interpolation

Wolfram-Validated Derivations Routed to General Relativity

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Abstract

This compendium assembles, in one rigorous document, (i) the complete classical theory of gravitation as Einstein's general relativity, with every curvature quantity derived and machine-verified; (ii) an account of *why* the observed particle zoo is large despite a small number of fundamental fields; (iii) a constructive catalogue of how soliton configurations are determined from spectral data and topological charge; and (iv) the connecting tissue of the *Unified Ontology*—a spectral/interpolation program in which orbital angular momentum ℓ is the organizing quantum number that maps eigenvalue ladders (atomic, nuclear, hadronic, and "musical" overtone series) onto one another. All symbolic and numerical results in boxed panels were computed and checked in the Wolfram Language; the residuals of every field equation are reported. The document closes with an explicit falsifiability ledger: the quantitative predictions by which the framework can be refuted.

Contents

Epistemic status. The general relativity, Standard Model, spectral theory, and soliton results in this report are established physics and are machine-verified here. The "Unified Ontology" synthesis—the proposal that a single spectral/interpolation principle indexed by orbital angular momentum organizes gravitation and the particle spectrum—is a *research program*, not a confirmed theory. No anomaly-free, divergence-free quantum unification of gravity with the Standard Model currently exists; that is physics' central open problem. Conjectural statements are derived from explicit postulates and flagged as such, and Section 8 states what would falsify them.

1 Foundations and Notation

We work in 3+1 dimensions with signature $(-,+,+,+)$. Greek indices run $0\ldots3$. Geometric units $G = c = 1$ are used for curvature computations and restored explicitly for physical predictions. $\hbar$ is kept where quantum spectra appear. The central objects are:

- a Lorentzian metric $g_{\mu\nu}$ and its Levi-Civita connection $\Gamma^{\alpha}{}_{\mu\nu}$;
- self-adjoint operators $\hat{H}$ (Schrödinger / Laplace–Beltrami / Dirac) whose spectra $\{\lambda_n\}$ encode matter;
- field configurations $\phi(x)$ of finite energy that are *solitons*—classically stable, particle-like lumps classified by topological charge.

Definition 1 (Unified Ontology, working definition)**.** *The* Unified Ontology *(UO) is the hypothesis that the physically realized states of matter and geometry are the discrete spectrum of a single family of self-adjoint operators $\hat{H}_\ell$ graded by orbital angular momentum $\ell \in \mathbb{Z}_{\geq 0}$, together with an* interpolation map $\mathcal{I}$ *that continuously deforms one sector's spectrum into another (atomic $\leftrightarrow$ hadronic $\leftrightarrow$ geometric) while preserving the Casimir $\ell(\ell+1)$ that also appears in the centrifugal term of the geodesic equation of general relativity.*

2 The Complete Theory of Gravity

2.1 The field equations

General relativity is defined by the Einstein–Hilbert action

$$S = \frac{c^4}{16\pi G}\int R\sqrt{-g}\,d^4x \;+\; S_{\text{matter}}, \tag{1}$$

whose stationary point under $\delta g^{\mu\nu}$ yields the field equations

$$G_{\mu\nu} \equiv R_{\mu\nu} - \tfrac{1}{2}R\,g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}\,T_{\mu\nu}. \tag{2}$$

Here $R_{\mu\nu}$ is the Ricci tensor, R the Ricci scalar, Λ the cosmological constant, and $T_{\mu\nu}$ the stress–energy of matter. The connection and curvature are built hierarchically:

$$\Gamma^{\alpha}{}_{\mu\nu} = \tfrac{1}{2} g^{\alpha\sigma}\big(\partial_\mu g_{\sigma\nu} + \partial_\nu g_{\sigma\mu} - \partial_\sigma g_{\mu\nu}\big), \tag{3}$$

$$R^{\rho}{}_{\sigma\mu\nu} = \partial_\mu \Gamma^{\rho}{}_{\nu\sigma} - \partial_\nu \Gamma^{\rho}{}_{\mu\sigma} + \Gamma^{\rho}{}_{\mu\lambda}\Gamma^{\lambda}{}_{\nu\sigma} - \Gamma^{\rho}{}_{\nu\lambda}\Gamma^{\lambda}{}_{\mu\sigma}. \tag{4}$$

The contracted Bianchi identity $\nabla^\mu G_{\mu\nu} = 0$ guarantees local covariant energy–momentum conservation $\nabla^\mu T_{\mu\nu} = 0$, an *identity* of the geometry rather than an extra assumption—the structural reason the theory is free of the most common consistency anomalies at the classical level.

2.2 The Schwarzschild solution, verified

For a static, spherically symmetric vacuum, $T_{\mu\nu} = 0$, the unique solution (Birkhoff's theorem) is

$$ds^2 = -\Big(1 - \tfrac{2M}{r}\Big)c^2 dt^2 + \Big(1 - \tfrac{2M}{r}\Big)^{-1} dr^2 + r^2\big(d\theta^2 + \sin^2\theta\, d\phi^2\big). \tag{5}$$

We computed the full curvature hierarchy from this metric symbolically.

Wolfram Language — Schwarzschild curvature (verified)

Computing Γ, $R^{\rho}{}_{\sigma\mu\nu}$, $R_{\mu\nu}$, R, $G_{\mu\nu}$, and the Kretschmann scalar $K = R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}$:

$$R = 0, \qquad G_{\mu\nu} = 0 \ \text{(all components)}, \qquad \boxed{K = \frac{48\,M^2}{r^6}}.$$

The vanishing Einstein tensor confirms the vacuum field equations are satisfied *identically*; the finite, r-dependent K shows the horizon $r = 2M$ is a coordinate artifact while $r = 0$ is a true curvature singularity.

2.3 Orbital angular momentum in the geodesic equation

The geodesic Lagrangian for the Schwarzschild metric admits two Killing constants: energy E and angular momentum L. The radial motion reduces to an effective one-dimensional problem,

$$\Big(\frac{dr}{d\tau}\Big)^2 = E^2 - \Big(1 - \frac{2M}{r}\Big)\Big(1 + \frac{L^2}{r^2}\Big), \tag{6}$$

in which the term L^2/r^2 is the exact relativistic descendant of the Newtonian centrifugal barrier. The same combination $\ell(\ell{+}1)\hbar^2$ governs the angular sector of every central-force quantum problem (Sec. 4). This shared appearance of orbital angular momentum—as L^2/r^2 in geometry and as $\ell(\ell+1)$ in spectra—is the empirical hook on which the Unified Ontology hangs its interpolation map.

2.4 Classical test: perihelion precession

The extra GR term in (6) produces a non-closing orbit. The per-orbit precession is $\Delta\varpi = 6\pi GM/[c^2 a(1-e^2)]$.

Wolfram Language — Mercury perihelion (verified)

With $M = M_\odot$, $a = 5.79 \times 10^{10}\,\mathrm{m}$, $e = 0.2056$, $T = 87.969\,\mathrm{d}$:

$$\Delta\varpi = \boxed{42.996\ \text{arcsec/century}},$$

matching the observed anomalous precession ($\approx 43''$) to three figures.

This is the prototype of the report's method: a quantity is derived symbolically, evaluated numerically in Wolfram, and confronted with data.

3 Why So Many Subatomic Particles?

3.1 Few fields, many states

The apparent multiplicity of "hundreds of particles" is not a multiplicity of *fundamental* objects. The Standard Model is built from a small field content; the large catalogue is what *combinatorics*, *confinement*, and *excitation* do to those few fields.

Wolfram Language — Standard Model bookkeeping (verified)

Quark degrees of freedom	6 flavors $\times$ 3 colors $\times$ 2 (particle/anti) $= 36$
Lepton degrees of freedom	$6 \times 2 = 12$
Gauge bosons	8 gluons $+ W^{\pm}, Z\,(3) + \gamma\,(1) = 12$
Scalar	1 Higgs

From roughly two dozen fundamental fields, the observed hadron spectrum is generated by three mechanisms.

(1) Confinement + flavor combinatorics. Color confinement forbids free quarks; only color singlets propagate. Counting flavor-symmetric combinations gives the multiplets directly.

Wolfram Language — hadron multiplicities (verified)

Mesons ($q\bar{q}$ flavor combinations among 6 flavors): $6^2 = 36$ (the SU(6) $35 \oplus 1$). Baryons (symmetric 3-quark combinations): $\binom{6+3-1}{3} = \boxed{56}$, exactly the celebrated SU(6) **56-plet**. Restricting to the light u, d, s flavors reproduces the SU(3) octet and decuplet.

(2) Orbital and radial excitation. Each $q\bar{q}$ or qqq ground state spawns a tower of excitations labeled by orbital angular momentum ℓ and radial node number n—the same quantum numbers as atomic spectroscopy. This is where the "hundreds" come from: a finite set of flavor multiplets, each multiplied by an (n, ℓ) Regge tower.

(3) Generations. Three copies of the fermion content (the origin of which is unexplained in the Standard Model) triple the fundamental count.

UO reading (conjectural). In the Unified Ontology the multiplicities $1, 8, 10, 27, 56, \ldots$ are not independent inputs but *values of a single spectral-degeneracy function* $d(\ell, n)$ evaluated on the symmetric-group/Lie representations selected by confinement. The claim is that the *same* $d(\ell, n)$ controls atomic shell filling, nuclear magic numbers, and hadron multiplets under the interpolation map $\mathcal{I}$. This is the program's load-bearing conjecture; Sec. 8 gives its test.

4 Spectral Theory and Orbital Angular Momentum

4.1 The angular Casimir

On the sphere the Laplace–Beltrami operator separates, and the spherical harmonics Y_ℓ^m are its eigenfunctions. We verified the eigenvalues directly.

Wolfram Language — angular momentum eigenvalue equation (verified)

Acting with $-\left[\frac{1}{\sin\theta}\partial_\theta(\sin\theta\,\partial_\theta) + \frac{1}{\sin^2\theta}\partial_\phi^2\right]$ on Y_ℓ^m returns the eigenvalues

$$\ell = 0, 1, 2, 3, 4 \;\Rightarrow\; \ell(\ell+1) = \boxed{0,\ 2,\ 6,\ 12,\ 20.}$$

These are precisely the coefficients of the centrifugal term in the GR geodesic equation (6), the link the UO exploits.

4.2 The hydrogen ladder

The Coulomb problem yields the bound spectrum $E_n = -13.6057/n^2\,\text{eV}$ with degeneracy n^2 (summing $\ell = 0 \ldots n-1$, each $2\ell+1$-fold).

Wolfram Language — hydrogen spectrum (verified)

$$E_n\,(\text{eV}) = -13.606,\ -3.401,\ -1.512,\ -0.850,\ -0.544,\ -0.378 \quad (n = 1 \ldots 6).$$

4.3 The spectral–"musical" correspondence

The Unified Ontology's invocation of music theory is the observation that a physical system's timbre is its operator spectrum. Two cases bracket the idea.

Wolfram Language — overtone series vs. drumhead modes (verified)

1-D string (self-adjoint $-d^2/dx^2$ on $[0, L]$): overtone ratios $1, 2, 3, 4, 5, 6$ (the harmonic series, hence consonant intervals).
Circular membrane (2-D Laplacian, Bessel zeros $j_{0,k}$): mode ratios 1, 2.295, 3.598, 4.903, 6.209, 7.515—*inharmonic*.

The lesson is structural: *geometry dictates spectrum dictates "harmony."* The integer harmonic series is special to one-dimensional self-adjoint operators; higher-dimensional or curved domains generically give inharmonic spectra, just as hadronic and nuclear spectra are inharmonic. Equal temperament is itself an *interpolation*—a logarithmic rounding of just ratios onto $2^{k/12}$.

Wolfram Language — 12-TET vs. just intonation (verified)

Largest cent deviations of 12-tone equal temperament from just ratios: major third $-13.7\,¢$, minor sixth $+13.7\,¢$, minor third $+15.6\,¢$, major sixth $-15.6\,¢$; perfect fifth only $+1.96\,¢$. Equal temperament is the optimal interpolation $\mathcal{I}$ that distributes the syntonic/Pythagorean commas across the octave.

UO reading (conjectural). The interpolation map $\mathcal{I}$ is modeled on equal temperament: a spectrum-preserving logarithmic rationalization. The conjecture is that the physical sectors (atomic, nuclear, hadronic) are "temperaments" of one underlying ℓ-graded operator, related by $\mathcal{I}$ much as keys are related on a tempered instrument. This is an analogy promoted to an ansatz; it earns its keep only through the quantitative test in Sec. 8.

5 Soliton Configurations: A Constructive Catalogue

5.1 What a soliton is

A soliton is a finite-energy, non-dispersive, classically stable solution of a nonlinear field equation, stabilized by a balance between dispersion and nonlinearity and—when topological—protected by a conserved winding number. Particles, in the soliton picture, *are* these lumps; their quantum numbers are spectral (the fluctuation operator's eigenvalues) and topological (the homotopy class).

5.2 The sine-Gordon kink, verified

The model $\partial_t^2\phi - \partial_x^2\phi + \sin\phi = 0$ has the static kink $\phi(x) = 4\arctan e^x$.

Wolfram Language — sine-Gordon kink (verified)

Equation-of-motion residual $\phi'' - \sin\phi$ evaluated at $x = -2, -1, 0, 1, 2$ returns $\{1.1 \times 10^{-16}, 0, 0, 0, -1.1 \times 10^{-16}\}$ (machine zero).
Rest energy $E = \int [\frac{1}{2}\phi'^2 + (1 - \cos\phi)]dx = \boxed{8}$ (natural units).
Topological charge $Q = \frac{1}{2\pi}[\phi(\infty) - \phi(-\infty)] = \boxed{1}$.

The integer charge Q is the prototype of a conserved "particle number" arising purely from boundary topology, not from a symmetry of the Lagrangian.

5.3 The KdV soliton, verified

The Korteweg–de Vries equation $u_t + 6uu_x + u_{xxx} = 0$ has the one-soliton

$$u(x,t) = \tfrac{c}{2}\,\mathrm{sech}^2\Big[\tfrac{\sqrt{c}}{2}(x - ct)\Big].$$

Wolfram Language — KdV soliton (verified)

Substituting into $u_t + 6uu_x + u_{xxx}$ yields residual $\equiv 0$ for all c: an *exact* solution. The amplitude $c/2$ and speed c are locked together—taller solitons travel faster—the hallmark of integrable nonlinearity.

5.4 How to determine the configurations: the recipe

For a given field theory, the soliton spectrum is found by a fixed, falsifiable procedure—the operational core of "determining the hundreds of configurations."

1. **Topological sector.** Compute the relevant homotopy group of the vacuum manifold (π_0 for kinks/domain walls, π_1 for vortices/strings, π_2 for monopoles, π_3 for Skyrmions/instantons). Its elements label the conserved charges Q.
2. **Bogomolny bound.** Where the energy admits a completion of squares, $E \geq |Q| \cdot (\text{const})$, the BPS configurations saturating it are the stable ground states of each sector.
3. **Fluctuation spectrum.** Linearize about the soliton; the self-adjoint fluctuation operator's discrete eigenvalues give the soliton's internal excitation modes (its "ℓ, n" tower)—and any negative eigenvalue signals instability.
4. **Semiclassical quantization.** Collective coordinates (position, isospin orientation) are quantized; this is how the Skyrme model recovers the nucleon, Δ, and the baryon multiplets as rotational/vibrational states of a single classical lump.

UO reading (conjectural). The Unified Ontology proposes that the "hundreds of subatomic configurations" are the soliton sectors of one ℓ-graded field theory, enumerated by the product of (a) a topological index and (b) the degeneracy function $d(\ell, n)$ of Sec. 3. The Skyrme model is the existing, partially successful instance: baryons as topological solitons of the pion field with $Q =$ baryon number. The UO conjecture is that the *same* construction, interpolated by $\mathcal{I}$ across sectors, also yields the mesonic and excited towers without new free parameters. This is testable and currently *unproven*.

6 The Hierarchy, Anomalies, Divergences

A unified framework must confront three notorious obstacles. We state them quantitatively rather than asserting they are solved.

6.1 The hierarchy: why gravity is weak

Wolfram Language — coupling strengths (verified)

$\alpha_{\mathrm{EM}} = e^2/4\pi\varepsilon_0\hbar c = 7.297 \times 10^{-3} = 1/137.0$.
Gravitational analogue for two protons $\alpha_G = Gm_p^2/\hbar c = 5.91 \times 10^{-39}$.
Ratio (electromagnetic / gravitational strength between two protons) = $\boxed{1.24 \times 10^{36}}$.

This 10^{36} gap is the hierarchy problem. No framework, the UO included, is credible unless it either explains this number or shows it to be a free boundary condition. The UO offers *no* derivation of it at present; this is honestly recorded as an open deficit.

6.2 Anomalies

Quantum gauge anomalies must cancel or the theory is inconsistent. In the Standard Model they cancel *generation by generation* because $\sum_{\text{fermions}} Q_{\text{electric}} = 0$ within each generation ($3 \times (\frac{2}{3} - \frac{1}{3}) + (0 - 1) = 0$, with the factor 3 from color). Any extension—including a UO completion—must preserve this cancellation; it is a hard constraint, not a stylistic preference.

6.3 Divergences

General relativity quantized naively is perturbatively non-renormalizable: loop divergences proliferate without bound because G carries negative mass dimension. This is the precise sense in which "a divergence-free quantum theory of gravity" does not yet exist. Candidate resolutions (asymptotic safety, string theory, loop quantization) remain unconfirmed. The UO does not resolve this; it inherits the problem and must eventually address it to claim completeness.

7 Postulates of the Unified Ontology

We collect the program's assumptions explicitly, so they can be attacked.

Postulate 1 (Spectral primacy). *Physical states are the discrete spectrum of a family $\{\hat{H}_\ell\}$ of self-adjoint operators graded by orbital angular momentum ℓ.*

Postulate 2 (Geometric routing). *The Casimir $\ell(\ell+1)$ entering $\hat{H}_\ell$ is identical to the centrifugal coefficient in the general-relativistic geodesic equation (6); matter spectra and geodesic motion share this invariant.*

Postulate 3 (Interpolation). *There exists a spectrum map $\mathcal{I}$, modeled on equal-temperament rationalization, deforming one sector's spectrum into another while preserving topological charge and the degeneracy function $d(\ell, n)$.*

Postulate 4 (Soliton realization). *Stable particles are topological solitons of the ℓ-graded theory; their quantum numbers are (homotopy charge) $\times$ (fluctuation spectrum).*

8 Falsifiability Ledger

A framework that cannot fail is not physics. The UO commits to the following quantitative tests; failure of any one falsifies the corresponding postulate.

1. **Degeneracy universality.** If a single $d(\ell, n)$ truly underlies atomic shells, nuclear magic numbers, and hadron multiplets, then the magic numbers $2, 8, 20, 28, 50, 82, 126$ must be reproduced by the *same* function that gives the SU(6) 56-plet. A single mismatch (e.g. failing to predict 126) refutes Postulate 1/3.

2. **Interpolation prediction.** $\mathcal{I}$ must map the measured charmonium/bottomonium ℓ-tower onto the positronium ℓ-tower with *no* new parameters beyond a single scale. A predicted level off by more than experimental error refutes Postulate 3.
3. **Soliton mass relation.** The Bogomolny-saturated mass of the baryon soliton must match the nucleon to within the Skyrme model's known accuracy *and* extend to the Δ spacing; a wrong sign of the N–Δ splitting refutes Postulate 4.
4. **Geometric routing.** Any system where matter $\ell(\ell+1)$ and geodesic L^2/r^2 can be independently measured (e.g. frame-dragging on a quantized orbital) must show the same Casimir; a discrepancy refutes Postulate 2.
5. **Hierarchy honesty.** The framework predicts α_G/α_{EM}; if it cannot reproduce $\sim 10^{-36}$ without fine-tuning, it is incomplete (an acknowledged current failure, not yet a refutation).

9 Summary Table of Verified Results

Quantity	Wolfram result	Status
Schwarzschild Einstein tensor	$G_{\mu\nu} = 0$	vacuum eqs. satisfied
Kretschmann scalar	$48M^2/r^6$	singularity at $r = 0$ only
Mercury precession	$42.996''$/century	matches $\sim 43''$ obs.
$\hat{L}^2$ eigenvalues	$0, 2, 6, 12, 20$	$= \ell(\ell+1)$
Hydrogen levels	$-13.6/n^2$ eV	exact
SU(6) baryon multiplet	56	matches known 56-plet
sine-Gordon kink energy / charge	8 / $Q = 1$	exact soliton
KdV soliton residual	0	exact soliton
α_{EM}	$1/137.0$	matches
EM/grav strength	1.24×10^{36}	hierarchy gap

Table 1: Every entry computed and checked in the Wolfram Language.

10 Conclusion

The established pillars—Einstein's gravity, the Standard Model's economical field content, the spectral theory of angular momentum, and the soliton picture of particles—are mutually consistent and, as shown, machine-verifiable to high precision. The Unified Ontology proposes to bind them with one organizing principle: an orbital-angular-momentum-graded spectral operator joined across sectors by an interpolation map and realized physically as topological solitons. That synthesis is, at present, a structured conjecture with explicit postulates and explicit ways to die. It does not yet solve the hierarchy problem or the non-renormalizability of quantum gravity, and it should not be represented as having done so. Its value is as a falsifiable research program: the predictions in Sec. 8 are concrete enough to be tested and discarded if wrong, which is the only honest criterion a theory of everything can be held to.